Relativistic Radiative Flow in a Luminous Disk

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Abstract

Radiatively driven transfer flow perpendicular to a luminous disk was examined under a fully special relativistic treatment, taking into account radiation transfer. The flow was assumed to be vertical, and the gravity, the gas pressure, and the viscous heating were ignored. In order to construct the boundary condition at the flow top, the magic speed above the flat source was re-examined, and it was found that the magic speed above a moving source can exceed that above a static source ($\sim 0.45\ c$). Then, the radiatively driven flow in a luminous disk was numerically solved, from the flow base (disk "inside"), where the flow speed is zero, to the flow top (disk "surface"), where the optical depth is zero. For a given optical depth and appropriate initial conditions at the flow base, where the flow starts, a loaded mass in the flow was obtained as an eigenvalue of the boundary condition at the flow top. Furthermore, a loaded mass and the flow final speed at the flow top were obtained as a function of the radiation pressure at the flow base; the flow final speed increases as the loaded mass decreases. Moreover, the flow velocity and radiation fields along the flow were obtained as a function of the optical depth. Within the present treatment, the flow three velocity v is restricted to be within the range of $v < c/\sqrt{3}$, which is the relativistic sound speed, due to the relativistic effect.

Key words: accretion, accretion disks — astrophysical jets — radiative transfer — relativity — X-rays: stars

1. Introduction

Accretion disks are tremendous energy sources in the active universe (see Kato et al. 1998 for a review). In particular, when the mass-accretion rate highly exceeds the critical rate, the disk local luminosity exceeds the Eddington one, and mass loss from the disk surface driven by radiation pressure takes place.

Such a radiatively driven outflow from a luminous disk has been extensively studied in the context of models for astrophysical jets by many researchers (Bisnovatyi-Kogan, Blinnikov 1977; Katz 1980; Icke 1980; Melia, Königl 1989; Misra, Melia 1993; Tajima, Fukue 1996, 1998; Watarai, Fukue 1999; Hirai, Fukue 2001; Fukue et al. 2001; Orihara, Fukue 2003), as on-axis jets (Icke 1989; Sikora et al. 1996; Renaud, Henri 1998; Luo, Protheroe 1999; Fukue 2005a), as outflows confined by a gaseous torus (Lynden-Bell 1978; Davidson, McCray 1980; Sikora, Wilson 1981; Fukue 1982), or as jets confined by the outer flow or corona (Sol et al. 1989; Marcowith et al. 1995; Fukue 1999), and as numerical simulations (Eggum et al. 1985, 1988). In almost all of these studies, however, the disk radiation fields were treated as external fields, and the radiation transfer was not solved.

The radiation transfer in the disk, on the other hand, was investigated in relation to the structure of a static disk atmosphere and the spectral energy distribution from the disk surface (e.g., Meyer, Meyer-Hofmeister 1982; Cannizzo, Wheeler 1984; Shaviv, Wehrse 1986; Adam et al. 1988; Hubeny 1990; Mineshige, Wood 1990; Ross et

al. 1992; Artemova et al. 1996; Hubeny, Hubeny 1997, 1998; Hubeny et al. 2000, 2001; Davis et al. 2005; Hui et al. 2005). In these studies, however, the vertical movement and mass loss were not considered. Moreover, their treatments were restricted in the non-relativistic regime, and the relativistic effects were not considered.

In order to break such a situation, where radiation transfer has not been considered in the radiatively driven wind from the disk, we recently examined the radiatively driven vertical outflow – *moving photosphere* – in a luminous flat disk (Fukue 2005b, c). In these papers, the radiative transfer flow was analytically or numerically solved, but the obtained flow speed was limited in the subrelativistic regime, since the problem was treated up to the order of $(v/c)^1$. In some astrophysical jet sources, however, the jet speed is mildly or highly relativistic. In gamma-ray bursts, the flow speed is supposed to be extremely relativistic. Hence, in the next step, we should consider the transfer flow up to the full order of $(v/c)^2$ (cf. Fukue 1999; Hirai, Fukue 2001; Fukue et al. 2001; Orihara, Fukue 2003; Fukue 2005a without transfer). Moreover, although it was incorporated in the previous paper (Fukue 2005b), which were up to the first order of the flow velocity, the effect of radiation drag must become much more important (Phinney 1987; Icke 1989). Other relativistic effects would further appear in the fully relativistic regime.

In this paper, we thus examine the radiatively driven transfer outflow in a luminous flat disk within the fully relativistic regime of $(v/c)^2$. At the preliminary stage, we ignore the gravity of the central object, the gas pres-

sure, and the viscous heating, and we treat the wind as a vertical one-dimensional flow without rotational motion.

In the next section we describe basic equations in the vertical direction. In section 3 we examine the boundary condition at the flow top, and show the magic speed above the moving source. In section 4 we then solve and examine a radiative flow under the appropriate boundary conditions at the flow base and top. The final section is devoted to concluding remarks.

2.   Basic Equations

Let us suppose a luminous flat disk, deep inside which gravitational or nuclear energy is released via viscous heating or other processes. The radiation energy is transported in the vertical direction, and the disk gas, itself, also *moves* in the vertical direction due to the action of radiation pressure (i.e., plane-parallel approximation). For the sake of simplicity, in the present paper, the radiation field is considered to be sufficiently intense that both the gravitational field of, e.g., the central object and the gas pressure can be ignored: tenuous cold normal plasmas in the super-Eddington disk, cold pair plasmas in the sub-Eddington disk, or dusty plasmas in the sub-Eddington disk. Internal heating is also ignored: the flow in or near to the surface envelope of the disk. As for the order of the flow velocity v, we consider the fully special relativistic regime, where the terms are retained up to the second order of (v/c).

Under these assumptions, the radiation hydrodynamic equations for steady vertical (z) flows are described as follows (Kato et al. 1998).

The continuity equation is

$$\rho c u = J \ (= \text{const.}), \tag{1}$$

where ρ is the proper gas density, u the vertical four velocity, J the mass-loss rate per unit area, and c the speed of light. The four velocity u is related to the three velocity v by $u = \gamma v/c$, where γ is the Lorentz factor: $\gamma = \sqrt{1+u^2} = 1/\sqrt{1-(v/c)^2}$.

The equation of motion is

$$c^2 u \frac{du}{dz} = \frac{\kappa_{\text{abs}} + \kappa_{\text{sca}}}{c} \left[F\gamma(1+2u^2) - c(E+P)\gamma^2 u \right], \tag{2}$$

where κ_{abs} and κ_{sca} are the absorption and scattering opacities (gray), defined in the comoving frame, E the radiation energy density, F the radiative flux, and P the radiation pressure observed in the inertial frame. The first term in the brackets on the right-hand side of equation (2) means the radiatively driven force, which is modified to the order of u^2, whereas the second term is the radiation drag force, which is also modified, but roughly proportional to the velocity.

In the no-gas pressure approximation and without heating, the energy equation is reduced to a radiative equilibrium relation,

$$0 = j - c\kappa_{\text{abs}}E\gamma^2 - c\kappa_{\text{abs}}Pu^2 + 2\kappa_{\text{abs}}F\gamma u, \tag{3}$$

where j is the emissivity defined in the comoving frame.

In this equation (3), the third and fourth terms on the right-hand side appear in the relativistic regime.

For radiation fields, the zeroth-moment equation becomes

$$\frac{dF}{dz} = \rho\gamma \left[j - c\kappa_{\text{abs}}E + c\kappa_{\text{sca}}(E+P)u^2 \right.$$
$$\left. + \kappa_{\text{abs}}Fu/\gamma - \kappa_{\text{sca}}F(1+v^2/c^2)\gamma u \right]. \tag{4}$$

The first-moment equation is

$$\frac{dP}{dz} = \frac{\rho\gamma}{c} \left[ju/\gamma - \kappa_{\text{abs}}F + c\kappa_{\text{abs}}Pu/\gamma \right.$$
$$\left. - \kappa_{\text{sca}}F(1+2u^2) + c\kappa_{\text{sca}}(E+P)\gamma u \right]. \tag{5}$$

Finally, the closure relation *in the inertial frame* is

$$cP\left(1+\frac{2}{3}u^2\right) = cE\left(\frac{1}{3} - \frac{2}{3}u^2\right) + \frac{4}{3}F\gamma u. \tag{6}$$

As a closure relation, the usual Eddington approximation *in the comoving frame* is adopted. Radiative quantities are then transformed from the comoving frame to the inertial frame, and we have the closure relation (6) in the inertial frame (see Kato et al. 1998 for details).

Eliminating j with the help of equations (3), and using continuity equation (1), equations (2), (4), and (5) are rearranged as

$$cJ\frac{du}{dz} = (\kappa_{\text{abs}} + \kappa_{\text{sca}})\rho\frac{\gamma}{c} \left[F(1+2u^2) - c(E+P)\gamma u \right], \tag{7}$$

$$\frac{dF}{dz} = (\kappa_{\text{abs}} + \kappa_{\text{sca}})\rho u \left[c(E+P)\gamma u - F(1+2u^2) \right], \tag{8}$$

$$\frac{dP}{dz} = (\kappa_{\text{abs}} + \kappa_{\text{sca}})\rho\frac{\gamma}{c} \left[c(E+P)\gamma u - F(1+2u^2) \right]. \tag{9}$$

Integrating the sum of equations (7) and (9) yields to the momentum flux conservation along the flow,

$$cJu + P = K \ (= \text{const.}). \tag{10}$$

In the subrelativistic regime, this relation is reduced to that derived in Fukue (2005b). Similarly, after some manipulations, integrating the sum of equations (7) and (8) gives the energy flux conservation along the flow,

$$c^2 J\gamma + F = L \ (= \text{const.}). \tag{11}$$

In the subrelativistic regime, this relation means that the flux F is constant.

At this stage, the basic equations are the equation of motion (7), the mass flux (1), the momentum flux (10), the energy flux (11), and the closure relation (6).

Next, by introducing the optical depth τ by

$$d\tau = -(\kappa_{\text{abs}} + \kappa_{\text{sca}})\rho dz, \tag{12}$$

the equation of motion (7) is rewritten as

$$cJ\frac{du}{d\tau} = -\frac{\gamma}{c} \left[F(1+2u^2) - c(E+P)\gamma u \right]. \tag{13}$$

Furthermore, eliminating E with the help of equation (6), this equation (13) can be finally rewritten as

$$cJ\frac{du}{d\tau} = -\frac{\gamma}{c}\frac{F(1+4u^2) - 4cP\gamma u}{1-2u^2}, \tag{14}$$

or

$$c^2 J \gamma^2 \frac{d\beta}{d\tau} = -\frac{F(1+3\beta^2) - 4cP\beta}{1 - 3\beta^2}, \qquad (15)$$

where $\beta = v/c$.

We shall solve equations (14), (10), and (11) for appropriate boundary conditions. Before this, we discuss the boundary conditions at the flow top in the next section.

3. Magic Speed Above a Moving Photosphere

When there is no motion in a luminous flat disk ("static photosphere"), the radiation fields above the disk are easily obtained. Namely, just above the disk with surface intensity I_0, the radiation energy density E_s, the radiative flux F_s, and the radiation pressure P_s are $(2/c)\pi I_0$, πI_0, and $(2/3c)\pi I_0$, respectively, where subscript s denotes the values at the disk surface. In the problem of radiation transfer in the accretion disk, these values are usually adopted as boundary conditions (e.g., Artemova et al. 1996). As will be shown below, however, the radiation fields above the luminous disk are changed when the disk gas itself does move upward ("moving photosphere"). Even in such a case, however, if the flow speed is small compared with the speed of light, the conditions for a static photosphere would be approximately adopted, and we can use these conditions in a previous paper (Fukue 2005b), where the flow speed is limited in the subrelativistic regime. When the flow speed is of the order of the speed of light, on the other hand, we should carefully treat the boundary condition for the moving photosphere. Thus, in the present paper, where the flow is treated in a fully relativistic manner, we must derive the exact boundary conditions above the moving photosphere.

In addition, for the radiatively driven flow in the relativistic regime, it becomes important the effect of radiation drag, which suppresses the jet speed (Phinney 1987; Icke 1989). For the flow above the static photosphere without gravity, Icke (1989) found that the *magic speed* of jets becomes $[(4 - \sqrt{7})/3]\, c \sim 0.45\, c$. If the photophere is moving, however, such a magic speed will be also changed (cf. Fukue 2000).

Hence, before we can examine the relativistic radiative flow, we must derive the radiation fields above the moving photosphere and consider the boundary condition at the flow top (disk "surface").

Let us suppose the situation that a flat infinite photosphere is not static, but moving upward with a speed v_s ($= c\beta_s$, and the corresponding Lorentz factor is γ_s). Then, the direction and intensity of radiation are changed due to aberration and Doppler effects (cf. Kato et al. 1998; Fukue 2000).

The transformations of the photon frequency ν and photon direction θ between the inertial and comoving frames become

$$\frac{\nu_0}{\nu} = \gamma_s(1 - \beta_s \cos\theta) = \frac{1}{\gamma_s(1 + \beta_s \cos\theta_0)}, \qquad (16)$$
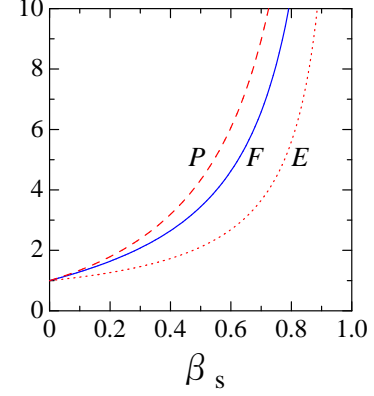
Fig. 1. Radiation fields above a moving photosphere with speed v_s ($= c\beta_s$). Each component of the radiation fields is normalized by the values of a static case. Every component increases as the speed increases.

$$\cos\theta = \frac{\cos\theta_0 + \beta_s}{1 + \beta_s \cos\theta_0}, \qquad (17)$$

where subscript 0 means the values measured in the comoving frame and $\gamma_s = 1/\sqrt{1 - \beta_s^2}$. For incident radiation with $\theta_0 = \pi/2$ in the comoving frame, the direction cosine in the inertial frame is $\cos\theta = \beta_s$.

Furthermore, the transformation of the intensity I between the inertial and comoving frames is

$$I_0 = \left(\frac{\nu_0}{\nu}\right)^4 I = [\gamma_s(1 - \beta_s \cos\theta)]^4 I$$
$$= \frac{1}{[\gamma_s(1 + \beta_s \cos\theta_0)]^4} I. \qquad (18)$$

Considering the Doppler effect (16) and aberration (17), the radiation fields above the moving photosphere in the inertial frame are calculated as follows:

$$cE_s = \int_0^{\cos^{-1}\beta_s} I\, d\Omega$$
$$= \frac{2\pi I_0}{\gamma_s^4} \int_0^{\cos^{-1}\beta_s} \frac{\sin\theta\, d\theta}{(1 - \beta_s \cos\theta)^4}$$
$$= 2\pi I_0 \gamma_s^2 \frac{3 + 3\beta_s + \beta_s^2}{3}$$
$$= 2\pi I_0 \frac{3\gamma_s^2 + 3\gamma_s u_s + u_s^2}{3}, \qquad (19)$$

$$F_s = \int_0^{\cos^{-1}\beta_s} I \cos\theta\, d\Omega$$
$$= \frac{2\pi I_0}{\gamma_s^4} \int_0^{\cos^{-1}\beta_s} \frac{\sin\theta \cos\theta\, d\theta}{(1 - \beta_s \cos\theta)^4}$$
$$= 2\pi I_0 \gamma_s^2 \frac{3 + 8\beta_s + 3\beta_s^2}{6}$$
$$= 2\pi I_0 \frac{3\gamma_s^2 + 8\gamma_s u_s + 3u_s^2}{6}, \qquad (20)$$
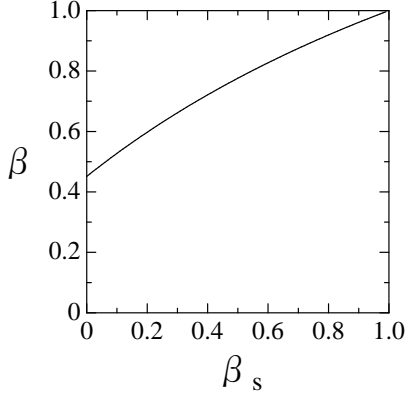
Fig. 2. Magic speed β above a moving photosphere as a function of the speed β_s of the photosphere (disk "surface"). When the photosphere is static, the magic speed is $\sim 0.45c$. When the photosphere is moving, however, the magic speed can exceed this limit.

$$
\begin{aligned}
cP_s &= \int_0^{\cos^{-1}\beta_s} I\cos^2\theta\, d\Omega \\
&= \frac{2\pi I_0}{\gamma_s^4}\int_0^{\cos^{-1}\beta_s}\frac{\sin\theta\cos^2\theta\, d\theta}{(1-\beta_s\cos\theta)^4} \\
&= 2\pi I_0\gamma_s^2\frac{1+3\beta_s+3\beta_s^2}{3} \\
&= 2\pi I_0\frac{\gamma_s^2+3\gamma_s u_s+3u_s^2}{3}. \quad (21)
\end{aligned}
$$

That is, the radiation fields depend on the speed v_s of the photosphere, and every component increases as the speed increases (see figure 1).

We must use these values of radiation fields as boundary conditions above the moving photosphere.

In addition, the magic speed above the luminous infinite disk is obtained by the condition where the radiative force is balanced with the radiation drag force. Hence, from equation (2), we have

$$
\begin{aligned}
0 &= F_s(1+2u^2)-c(E_s+P_s)\gamma u \\
&= [F_s(1+\beta^2)-c(E_s+P_s)\beta]\gamma^2, \quad (22)
\end{aligned}
$$

for the magic speed $\beta\ (=u/\gamma)$.

Inserting equations (19)–(21) into this equation (22), we can obtain the *magic speed above a moving photosphere* as a function of the speed β_s of the photosphere,

$$
\beta = \frac{4-\sqrt{7}+6\beta_s+(4+\sqrt{7})\beta_s^2}{3+8\beta_s+3\beta_s^2}. \quad (23)
$$

When the disk is static ($\beta_s=0$), this relation is reduced to that obtained by Icke (1989), $\beta=(4-\sqrt{7})/3\sim0.45$.

In figure 2, the magic speed above a moving photosphere is shown as a function of the speed of the photosphere. When the luminous photosphere is not static, but moving, the magic speed can exceed the limit of $\sim0.45c$, which Icke (1989) obtained for a static photosphere (cf. Fukue 2000).

4. Relativistic Radiative Transfer Flow

Now, we discuss our numerical solution of equations (14), (10), and (11) for appropriate boundary conditions.

4.1. Boundary Conditions and Singularity

As for boundary conditions, we impose the following cases. At the flow base (disk "inside") with an arbitrary optical depth τ_0 (which relates to the disk surface density), the flow velocity u is zero, the radiative flux is F_0 (which is a measure of the strength of radiation field), and the radiation pressure is P_0 (which connects with the radiation pressure gradient and relates to the disk internal structure), where subscript 0 denotes the values at the flow base. At the flow top (disk "surface") where the optical depth is zero, the radiation fields should satisfy the values above a moving photosphere derived in the previous section. Namely, just above the disk with surface intensity I_s, the radiation energy density E_s, the radiative flux F_s, and the radiation pressure P_s are, respectively,

$$
cE_s = 2\pi I_s\frac{3\gamma_s^2+3\gamma_s u_s+u_s^2}{3}, \quad (24)
$$

$$
F_s = 2\pi I_s\frac{3\gamma_s^2+8\gamma_s u_s+3u_s^2}{6}, \quad (25)
$$

$$
cP_s = 2\pi I_s\frac{\gamma_s^2+3\gamma_s u_s+3u_s^2}{3}, \quad (26)
$$

where $u_s\ (=\gamma_s v_s/c)$ is the flow four velocity at the flow top and subscript s denotes the values at the flow top.

Applying these boundary conditions to equations (10) and (11), we have two relations on the boundary values and mass-loss rate:

$$
Jc^2 u_s+cP_s = cP_0, \quad (27)
$$

$$
Jc^2\gamma_s+F_s = Jc^2+F_0. \quad (28)
$$

Physically speaking, in the radiative flow starting from the flow base with an arbitrary optical depth τ_0, for initial values of F_0 and P_0 at the flow base, the final values of the radiation fields E_s, F_s, P_s, and the flow velocity u_s at the flow top can be obtained by solving basic equations. Furthermore, the mass-loss rate J is determined as an eigenvalue so as to satisfy the bondary condition at the flow top (cf. Fukue 2005b in the subrelativistic regime).

In the present full relativistic case, however, the final values of the radiation fields at the flow top depend on the flow velocity there, and the final values at the flow top cannot be analytically expressed by the initial values at the flow base. Hence, in this paper we determine the mass-loss rate as follows.

In radiative flow with optical depth τ_0, we first give the final flow velocity u_s (and γ_s), instead of the initial value of P_0. Then, the final values of radiation fields E_s, F_s, and P_s can be fixed by equations (24)–(26). Next, we give a trial value for the mass-loss rate J, and the initial values of P_0 and F_0 can be fixed by equations (27) and (28). Since all the parameters are temporarily fixed, we solve equation (14) from $\tau=\tau_0$ to $\tau=0$. Generally, however, the obtained final velocity at $\tau=0$ is different from a given u_s. Thus,

we vary the value of J and follow iterative processes, so that the calculated final velocity coincides with a given final velocity u_s.

Another point to be noticed is the *singularity* in equation (14): i.e., the denominator of equation (14) vanishes when $u = \pm 1/\sqrt{2}$ or $\beta = \pm 1/\sqrt{3}$. This singularity originates from the closure relation (6), and coincides with the *sound speed* of the relativistic (photon) gas. In other words, equation (14) has a form of the transonic wind equation for the relativistic (photon) gas with sound speed $c/\sqrt{3}$. In usual wind equations from a gravitating source, there exist transonic (critical) points, where both the numerator and denominator of wind equations vanish simultaneously. In the present case, there also exist *critical points*, which yield that $u_c = \pm 1/\sqrt{2}$ ($\beta_c = \pm 1/\sqrt{3}$) and $P_c = \mathrm{sgn}(u)(\sqrt{3}/2)F_c$, where subscript c denotes the critical point. In addition, from the linear analysis around a critical point, the velocity gradient near to the singularity is found to be $du/d\tau|_c = \mathrm{sgn}(u)(3/4)$.

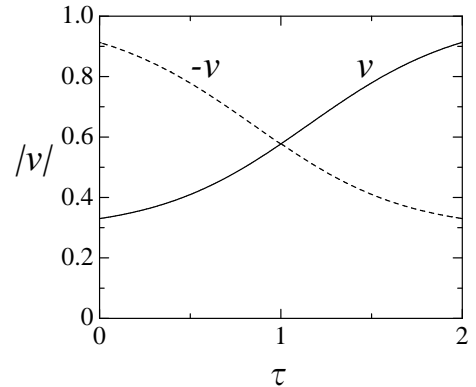We show an example of "critical solutions" in figure 3, where the parameters are $\tau_c = 1$, $F_c/(\pi I_s) = 1$, and $J/(\pi I_s/c^2) = 1$. As can be seen in figure 3, one of the critical solutions (a solid curve) has a positive velocity, and is an outward breeze solution, which decelerates toward low optical depth. Another (a dashed curve) has a negative velocity, and is an inward settling solution, which decelerates toward a high optical depth. Both solutions decelerate in the direction of the flow due to the radiation drag force. However, neither satisfies the present boundary conditions at the flow base and the flow top. Indeed, in order for flow to be accelerated, the velocity gradient $du/d\tau$ should always be negative, but it is positive around a critical point for an outward solution. Hence, the flow in the present framework cannot pass through the critical point. Thus, for the present purpose, such "critical solutions" are inadequate, and the flow is always subsonic or supersonic in the sense that the flow speed is always less than or greater than $c/\sqrt{3}$. And since we assume that the flow starts with $u = 0$ at $\tau = \tau_0$, we suppose a *subsonic* flow in the present paper: $u_s < 1/\sqrt{2} \sim 0.707$. We shall discuss this singularity problem later.

4.2. Subsonic Solutions

Examples of the results for relativistic radiative flows in a luminous disk under the present boundary conditions are shown in figures 4 and 5.

In figure 4 we show the final velocity v_s at the flow top (solid curves), the radiative flux F_0 at the flow base (upper dashed ones), the radiative flux F_s at the flow top (lower dashed ones), the radiation pressure P_s at the flow top (dotted ones), and the mass-loss rate J (chain-dotted ones), as a function of P_0 for several values of τ_0 at the flow base. The quantities are normalized in units of c and πI_s. For example, the unit of J is $\pi I_s/c^2$.

As can be seen in figure 4, as the radiative flux increases, the final flow velocity at the flow top increases, but the mass-loss rate decreases. Moreover, as can be seen in figure 4, and similar to the subrelativistic case (Fukue 2005b,


Fig. 3. Example of "critical solutions". The parameters are $\tau_c = 1$, $F_c/(\pi I_s) = 1$, and $J/(\pi I_s/c^2) = 1$. These critical solutions pass through the critical points of the present problem, but do not satisfy the present boundary conditions.

c), in order for flow to exist, the radiation pressure P_0 at the flow base is restricted to be within some range. In the subrelativistic case without gravity and heating (Fukue 2005b), the initial pressure P_0 is proved to be restricted within the range of $2/3 < cP_0/F_s < 2/3 + \tau_0$. In the present case, the initial pressure is also restricted to be within a similar range, but somewhat modified due to the relativistic effect: i.e., the radiative flux F_s is no longer constant, but depends on the flow final speed. At the one limit of P_0, the pressure gradient between the flow base and the top is maximum, the loaded mass diverges, and the flow final speed becomes zero. In the other limit of P_0, the pressure gradient vanishes, the loaded mass becomes zero, and the flow final speed becomes high.

As already stated, in the present model, the flow final speed is supposed to be within the range of $0 < u_s < 1/\sqrt{2}$ (i.e., $0 < v_s/c < 1/\sqrt{3}$). As a result, using equations (25) and (26), the radiative flux F_s and the radiation pressure P_s are restricted to be within the following ranges: $1 < F_s/(\pi I_s) < 4.31$ and $2/3 < cP_s/(\pi I_s) < 3.73$. Finally, although the flow final speed is less than the photon sound speed ($v_s/c < 1/\sqrt{3} \sim 0.577$), the flow final speed can exceed the magic speed above the luminous infinite disk [$v_{\mathrm{magic}}/c = (4 - \sqrt{7})/3 \sim 0.451$].

In figure 5 we show the flow three velocity v (solid curves), the radiative flux F (dashed curves), and the radiation pressure P (dotted curves) as a function of the optical depth τ for several values of u_s at the flow top in a few cases of τ_0. The quantities are normalized in units of c and πI_s.

When the initial radiative flux F_0 at the flow base is large, the flow is effectively accelerated, and the flow final speed becomes large. On the other hand, when the pressure gradient between the flow base and the flow top is large, the loaded mass J becomes large (cf. figure 4). The
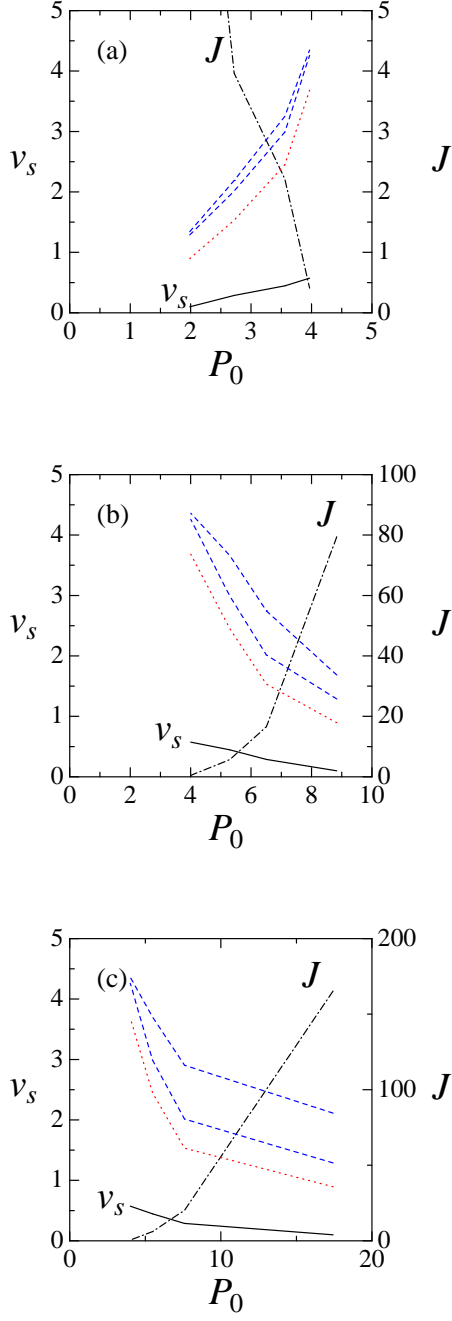
Fig. 4. Final velocity v_s at the flow top (solid curves), the radiative flux F_0 at the flow base (upper dashed ones), the radiative flux F_s at the flow top (lower dashed ones), the radiation pressure P_s at the flow top (dotted ones), and the mass-loss rate J (chain-dotted ones), as a function of P_0 for several values of τ_0 at the flow base: (a) $\tau_0 = 1$, (b) $\tau_0 = 10$, and (c) $\tau_0 = 100$. The quantities are normalized in units of c and πI_s. That is, the unit of F and cP is πI_s and the unit of J is $\pi I_s/c^2$. The left-side scale of the ordinates is for quantities except for J, while the right-side is for J.
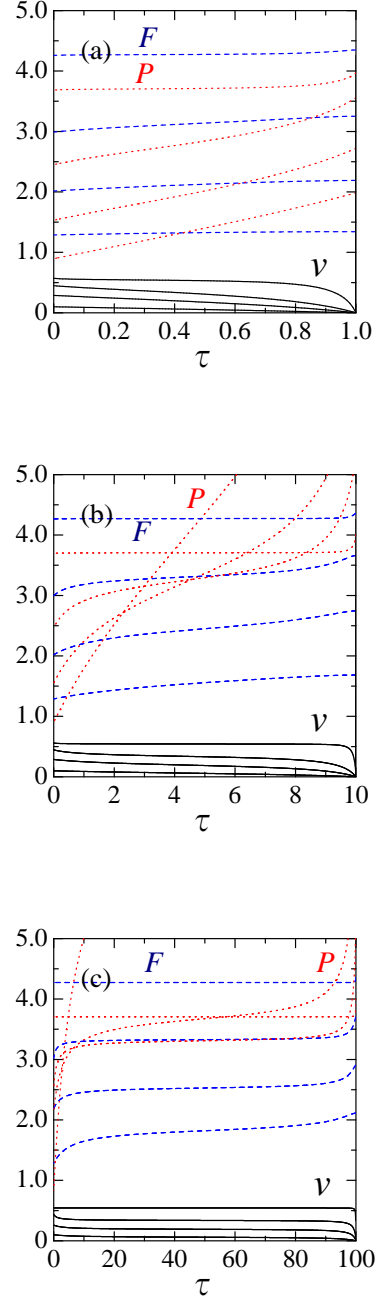
Fig. 5. Flow three velocity v (solid curves), radiative flux F (dashed curves), and radiation pressure P (dotted curves), as a function of the optical depth τ for several values of u_s at the flow top in a few cases of τ_0. The parameters are (a) $\tau_0 = 1$, (b) $\tau_0 = 10$, and (c) $\tau_0 = 100$, and from bottom to top of v, F, and P, the values of u_s are 0.1, 0.3, 0.5, and 0.7. The quantities are normalized in units of c and πI_s. That is, the unit of F and cP is πI_s.

latter properties are somewhat complicated, and should be explained much more.

When the optical depth τ_0 is not so large (figures 4a and 5a), the pressure gradient between the flow base and top is not so large. In this case, the pressure gradient becomes large, as the pressure P_0 at the flow base becomes small. This is the reason that the loaded mass becomes large, since P_0 is small (figure 4a). When the optical depth is large (figures 4b, 4c and 5b, 5c), on the other hand, the pressure gradient generally becomes large. Hence, the pressure gradient is large, as the pressure P_0 at the flow base becomes large. This is the reason that the loaded mass becomes large, since P_0 is large (figures 4b and 4c).

Alternatively, in the less-luminous, sub/non-relativistic limit with small $u_{\rm s}$, the radiation fields are roughly expressed as

$$F \sim F_{\rm s}, \tag{29}$$

$$cP \sim F_{\rm s}(2/3 + \tau), \tag{30}$$

which are the usual Milne approximations (cf. Fukue 2005b, c). Hence, in the less-luminous, small velocity case, the pressure gradient between the flow top and base becomes large, as the optical depth is large. In the highly-luminous, relativistic limit with large $u_{\rm s}$, on the other hand, the flow velocity quickly reaches the equilibrium velocity, where the right-hand side of equation (14) or (15) vanishes, and becomes constant as

$$\beta \sim \frac{2}{3}\frac{cP}{F} - \sqrt{\left(\frac{2}{3}\frac{cP}{F}\right)^2 - \frac{1}{3}}, \tag{31}$$

where the radiative flux F and the radiation pressure P are also approximately constant according to equations (10) and (11). For example, in the limiting case of $\beta = 1/\sqrt{3}$, $cP/F = \sqrt{3}/2$. As a result, the pressure gradient is small even for large optical depth.

In any case, the mass-loss rate J is not given arbitrarily, but is determined as an eigenvalue, similar to the subrelativistic case (Fukue 2005b).

5. Concluding Remarks

In this paper we have examined radiative flow in a luminous disk, while taking into account radiative transfer, in a fully relativistic manner (cf. Fukue 2005b, c for a subrelativistic regime). The vertical velocity v, the radiative flux F, and the radiation pressure P are numerically solved as a function of the optical depth τ for the cases without gravity and heating. At the flow base (disk "inside") where the flow speed is zero, the initial optical depth, the initial radiative flux, and the initial radiation pressure are τ_0, F_0, and P_0, respectively; in the usual accretion disk these quantities are determined in terms of the central mass, the mass-accretion rate, and the viscous process as a function of radius. At the flow top (disk "surface") where the optical depth τ is zero, the radiation fields ($E_{\rm s}$, $F_{\rm s}$, and $P_{\rm s}$) should coincide with those above a *moving* photosphere with the final speed $v_{\rm s}$ with uniform intensity. In order to match this boundary condition, the mass-loss rate J is determined as an eigenvalue.

One of the relativistic manifestations is this boundary condition at the flow top. The radiation fields are not for the static flat source, but should be for the moving flat source. As a result, it is found that the magic speed above a moving photosphere can exceed that above a static photosphere ($\sim 0.45\ c$).

Another relativistic manifestation is the existence of the singular point in the equation, which is related to the sound speed of the relativistic (photon) gas. Since the "critical solutions" for the present problem do not satisfy the present boundary conditions, we obtained *subsonic* solutions, which is always less than the relativistic sound speed of $c/\sqrt{3}$. Nevertheless, we can find the relativistic solution, whose final speed is greater than the magic speed above a static source.

The appearance of this singularity may come from the closure relation for the quantities of radiation fields, which assumes the usual Eddington approximation in the comoving frame (Kato et al. 1998). That is, in the comoving frame we assume

$$P_0^{ij} = \frac{\delta^{ij}}{3}E_0, \tag{32}$$

where subscript 0 denotes the quantities in the comoving frame. The appearance of the singularity suggests that the Eddington approximation would be violated in the relativistic flow, whose velocity is greater than $c/\sqrt{3}$. This may be because the diffusion speed in the comoving frame cannot exceed $c/\sqrt{3}$, and/or the diffusion would not be isotropic in the comoving frame. Indeed, if we assume that the diffusion in the comoving frame is no longer isotropic, but the factor $1/3$ in equation (32) is variable, similar to the usual variable Eddington factor, we can formally extinguish the singularity.

The plane-parallel assumption may also play a role in the existence of the singularity, as well as the existence of the radiation drag force, which is related to the closure approximation transformed to the inertial frame. The nature of critical points should be examined more carefully. In any case, the existence of the singularity does not mean that the flow velocity cannot exceed $c/\sqrt{3}$, but merely suggests a possible failure of the formalism adopted in the present analysis.

The relativistic radiative flow investigated in the present paper must be quite *fundamental* to accretion disk physics, astrophysical jet formation, gamma-ray bursts, etc., although the present paper is only the first step and there are many simplifications at the present stage.

At first, in a rigorous sense, the present situations, such as a plane-parallel assumption, are valid only in very specific circumstances. Indeed, when the accretion rate exceeds the critical one, the disk would puff up to be geometrically thick, and the plane-parallel approximation would be violated. In a rough sense, however, the present situations could be approximately valid in such a thick disk, as long as the gas motion and the radiative flux are almost vertical. If the gas motion is not vertical, but expands like a spherical flow, the present treatment must be reexam-

ined. In contrast to the plane-parallel case considered in the present paper, the spherically expanding case is also of great interest to us, and should be examined in the future.

In the present paper, we have assumed that the opacity is frequency-independent (gray), and therefore the radiation pressure force is independent of the velocity. In a cold gas, however, atomic absorption opacities become important. Such opacities are frequency-dependent, and therefore the radiation pressure force would be a function of the Doppler shift between the main absorption troughs and the peak of the black body spectrum from the disk. Hence, the radiation energy flux L would generally be a function of the velocity u. In addition, in such a case of atomic opacities, the super-Eddington radiation-pressure force on the flow may arise from a larger absorption opacity. Since the acceleration nature would be drastically changed, such a *line-driven radiative flow* must be carefully examined.

Furthermore, we ignored the gravitational field produced by the central object. This means that the flow considered in the present paper would correspond to normal plasmas in the super-Eddington disk, pair plasmas in the sub-Eddington disk, or dusty plasmas in the luminous disk. In other cases, or even for normal plasmas in the super-Eddington disk, the gravitational field would affect the flow properties (cf. Fukue 2005c for a subrelativistic case). In particular, since the gravitational field in the vertical direction is somewhat complicated (e.g., Fukue 2002, 2004), the influence of the gravitational field is important.

We also ignored the gas pressure. This means that the radiation field is sufficiently intense. In general cases, where the gas pressure is considered, there usually appear sonic points (e.g., Fukue 2002, 2004), and the flow is accelerated from subsonic to supersonic.

In the high-energy regime, including relativistic flow and hot gas, the Compton heating and cooling would be important, although we have dropped such an effect. If the Compton heating of the gas by intense radiation fields works, the gas might be radiatively heated to very high temperatures, and the atomic absorption opacity could be significantly reduced. Such a radiative effect in the energy equation should also be examined in future work.

Finally, it is left as an open question whether the singularity appearing in the present paper can be passed through or whether the relativistic radiation transfer formalism should be improved.

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